2812 [1920, 81]. Proposed by C. N. SCHMALL, New York City.

If $F(x, y, z)$ be a homogeneous function of x, y, z which becomes $\phi(u, v, w)$ by elimination of x, y, z by means of the equations, $\partial F/\partial x = u$, $\partial F/\partial y = v$, $\partial F/\partial z = w$; show that

$$\frac{\partial F}{\partial u}\Big/x = \frac{\partial F}{\partial v}\Big/y = \frac{\partial F}{\partial w}\Big/z.$$

SOLUTION BY THE PROPOSER.

The result is incorrectly stated. It should read,

$$\frac{\partial \phi}{\partial u}\Big/x = \frac{\partial \phi}{\partial v}\Big/y = \frac{\partial \phi}{\partial w}\Big/z.$$

Let k be the degree of the homogeneous function F. Then by Euler's theorem on homogeneous functions,

$$xu + yv + zw = kF. \tag{1}$$

By partial differentiation of (1) with respect to x, we obtain

$$u + x\frac{\partial u}{\partial x} + y\frac{\partial v}{dx} + z\frac{\partial w}{\partial x} = ku,$$

or

$$x\frac{\partial u}{\partial x} + y\frac{\partial v}{\partial x} + z\frac{\partial w}{\partial x} = (k-1)u. \tag{2}$$

Similarly, we get

$$x\frac{\partial u}{\partial y} + y\frac{\partial v}{\partial y} + z\frac{\partial w}{\partial y} = (k-1)v \tag{3}$$

and

$$x\frac{\partial u}{\partial z} + y\frac{\partial v}{\partial z} + z\frac{\partial w}{\partial z} = (k-1)w. \tag{4}$$

Again,

$$u = \frac{\partial F}{\partial x} = \frac{\partial \phi}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial \phi}{\partial v}\cdot\frac{\partial v}{\partial x} + \frac{\partial \phi}{\partial w}\cdot\frac{\partial w}{\partial x}, \tag{5}$$

$$v = \frac{\partial F}{\partial y} = \frac{\partial \phi}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial \phi}{\partial v}\cdot\frac{\partial v}{\partial y} + \frac{\partial \phi}{\partial w}\cdot\frac{\partial w}{\partial y}, \tag{6}$$

$$w = \frac{\partial F}{\partial z} = \frac{\partial \phi}{\partial u}\cdot\frac{\partial u}{\partial z} + \frac{\partial \phi}{\partial v}\cdot\frac{\partial v}{\partial z} + \frac{\partial \phi}{\partial w}\cdot\frac{\partial w}{\partial z}. \tag{7}$$

The solution of the system (2), (3), (4) for x, y, z, and of the system (5), (6), (7) for $\partial\phi/\partial u$, $\partial\phi/\partial v$, $\partial\phi/\partial w$ gives

$$\frac{\partial \phi}{\partial u}\Big/x = 1/(k-1), \qquad \frac{\partial \phi}{\partial v}\Big/y = 1/(k-1), \qquad \frac{\partial \phi}{\partial w}\Big/z = 1/(k-1).$$

Whence,

$$\frac{\partial \phi}{\partial u}\Big/x = \frac{\partial \phi}{\partial v}\Big/y = \frac{\partial \phi}{\partial w}\Big/z.$$

2823 [1920, 185]. Proposed by S. A. COREY, Des Moines, Iowa.

Let TQ and PR be diameters of a circle with center O. Bisect TO at X and draw PQ. On PQ erect the perpendicular XW and on PR, the perpendicular QV. Prove that $OX\cdot PV = PW\cdot PQ$

SOLUTION BY EMMA M. GIBSON, Springfield, Mo.

If S is the intersection of XW and PO, the triangle XOS is isosceles (the reader is requested to draw the figure); for,

$$\angle SXO = 90° - \angle XQW = 90° - \angle SPW = \angle PSW = \angle XSO.$$

Hence $OX = OS = SP$. Since the triangles PWS and PVQ are similar,

$$PW/PV = PS/PQ = OX/PQ,$$

and hence

$$OX\cdot PV = PW\cdot PQ.$$

Also solved by H. L. AGARD, T. M. BLAKSLEE, H. N. CARLETON, P. J. DA CUNHA, H. H. DOWNING, R. M. GINNINGS, C. E. MANGE, H. L. OLSON, LOUIS O'SHAUGHNESSY, ARTHUR PELLETIER, J. B. REYNOLDS, A. V. RICHARDSON, and J. H. WEAVER.

2838 [1920, 273–274].

"A rope is supposed to be hung over a wheel fixed to the roof of a building; at one end of the rope a weight is fixed, which exactly counterbalances a monkey which is hanging on to the other end. Suppose that the monkey begins to climb the rope, what will be the result?"

This problem was invented by Lewis Carroll in December, 1893 (S. D. Collingwood, *The Life and Letters of Lewis Carroll* (*Rev. C. L. Dodgson*), New York, 1899, pp. 317–318), and in his diary he remarked: "Got Professor Clifton's answer [R. B. Clifton, professor of physics at Oxford] to the 'Monkey and Weight Problem.' It is very curious, the different views taken by good mathematicians. Price [Bartholomew Price, professor of physics at Oxford] says that the weight goes *up* with increasing velocity; Clifton (and Harcourt [A. G. Vernon-Harcourt, professor of chemistry at Oxford]) that it goes *up*, at the same rate as the monkey; while Sampson [probably E. F. Sampson, lecturer, tutor and censor of Christ Church, Oxford] says that it goes *down*." Yet another solution by Rev. A. Brook is given on page 268 of *The Lewis Carroll Picture Book* . . . edited by S. D. Collingwood (London, 1899), namely, that "the weight remains stationary."

The problem has been recently discussed in *School Science and Mathematics*, volume 17, December, 1917, p. 821; volume 19, December, 1919, p. 815; and volume 20, February, 1920, pp. 172–173. The editors of the MONTHLY invite mathematical solutions of the problem.

The following solutions were contributed by request:

I. SOLUTION BY E. V. HUNTINGTON, Harvard University.

Case 1. If we neglect the weight of the pulley and rope, the solution follows immediately from the fundamental principle of mechanics, namely: the acceleration of a particle in any direction is proportional to the net force acting on the particle in that direction.

Here there are two particles to consider: (1) the monkey, and (2) the counterpoise.

The net upward force acting on the monkey is $T - W$, where W is the weight of the monkey, and T the tension in his part of the rope. The net upward force acting on the counterpoise is $T' - W'$, where W' is the weight of the counterpoise, and T' the tension in that part of the rope. But on the hypothesis of Case 1, the tension in the rope is the same at all points, so that $T' = T$; also, W' is known to be equal to W. Hence the net upward force acting on the monkey is the same as the net upward force acting on the counterweight, so that *the accelerations of the two bodies must be equal at every instant.*

Therefore, since the two bodies may be supposed to start from rest at the same level, their motions will be precisely parallel, no matter how fast or slow the monkey may climb, up or down, or how much he may allow the rope to slip through his hands.

Case 2. If we take into account the weight of the wheel (still neglecting the weight of the rope), we shall need to use also the equation of rotation.

Let w_0 be the weight of the wheel, r its radius, and k its radius of gyration. Then the equations of motion for the three bodies, W, W', and w_0, will be:

$$T - W = (W/g)dv/dt, \qquad T' - W' = (W'/g)dv'/dt,$$

and

$$Tr - T'r = (w_0/g)k^2 d\omega/dt,$$

where v and v' are the velocities of the monkey and the counterpoise, respectively, in the upward direction in space, and ω is the angular velocity of the wheel.

The geometric conditions of the problem tell us that as long as the rope does not become slack

$$v' = r\omega, \qquad \text{and} \qquad v' + v - u = 0,$$

where u is the relative velocity of the monkey up the rope.

From these five equations, we readily find:

$$\left(W + W' + \frac{w_0 k^2}{r^2}\right)\frac{dv'}{dt} = W\frac{du}{dt} + (W - W')g,$$